

DATE: _____

SUBJECT: See

Forward Kinematics. (unique solution)

$$\theta \rightarrow q_i \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow[\text{orientation}]{\text{position}} H_{EE} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

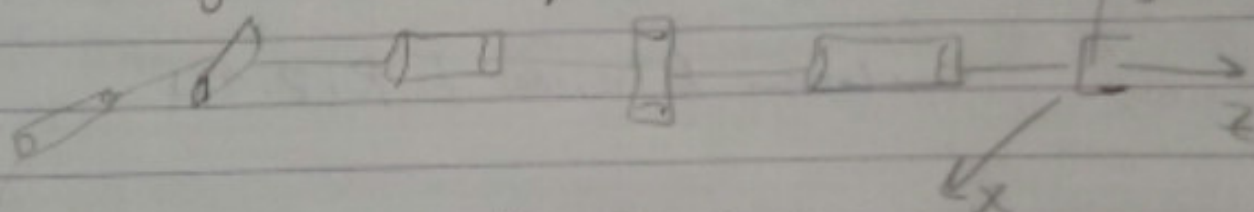
 $h_{ij} \rightarrow \text{function of } q_i(q_k) \quad k = 1, 2, \dots, n$

Inverse Kinematics (multiple solutions) $q_k = f_n(P_{des})$
 given desired pose

$$P_{des} \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow q_k$$

given H_{EE} desired
 to end effector.

model from solving 12 non linear eqns

 $h_{ij} \rightarrow \text{certain value}$ 

$$H_{EE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -12 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} P_{12}(q_k) \\ P_{12}E_{12} = 0 \end{matrix}$$

$$P_{24}(q_k) = -12$$

12 non linear equations

DOF

DOF

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دالة جاكوب الأصلية مع الأصل
 origin wrist center
 wrist center

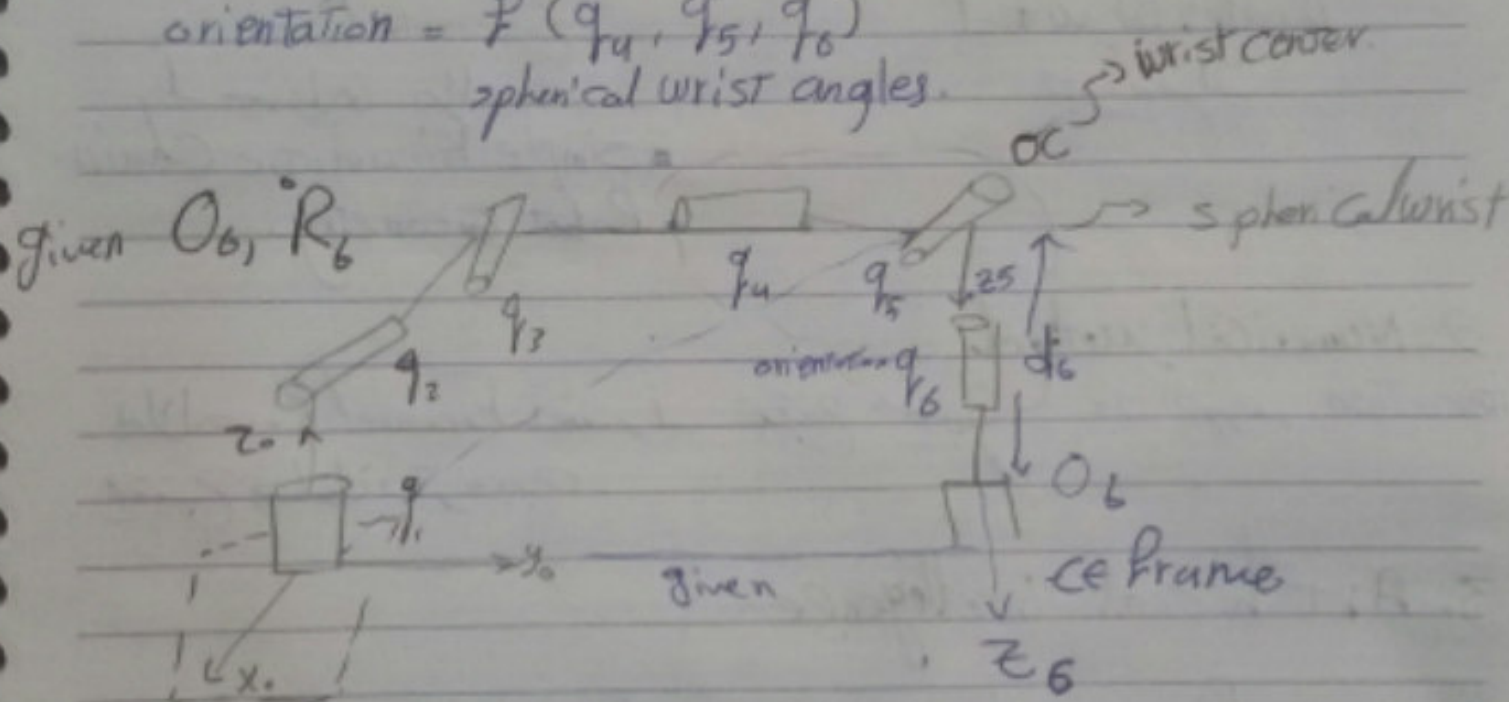
Kinematic Decoupling: Geometric approach

6 DOF manipulator
 spherical wrist

6 DOF Robot

wrist center = $f(q_1, q_2, q_3)$

orientation = $f(q_4, q_5, q_6)$
 spherical wrist angles.



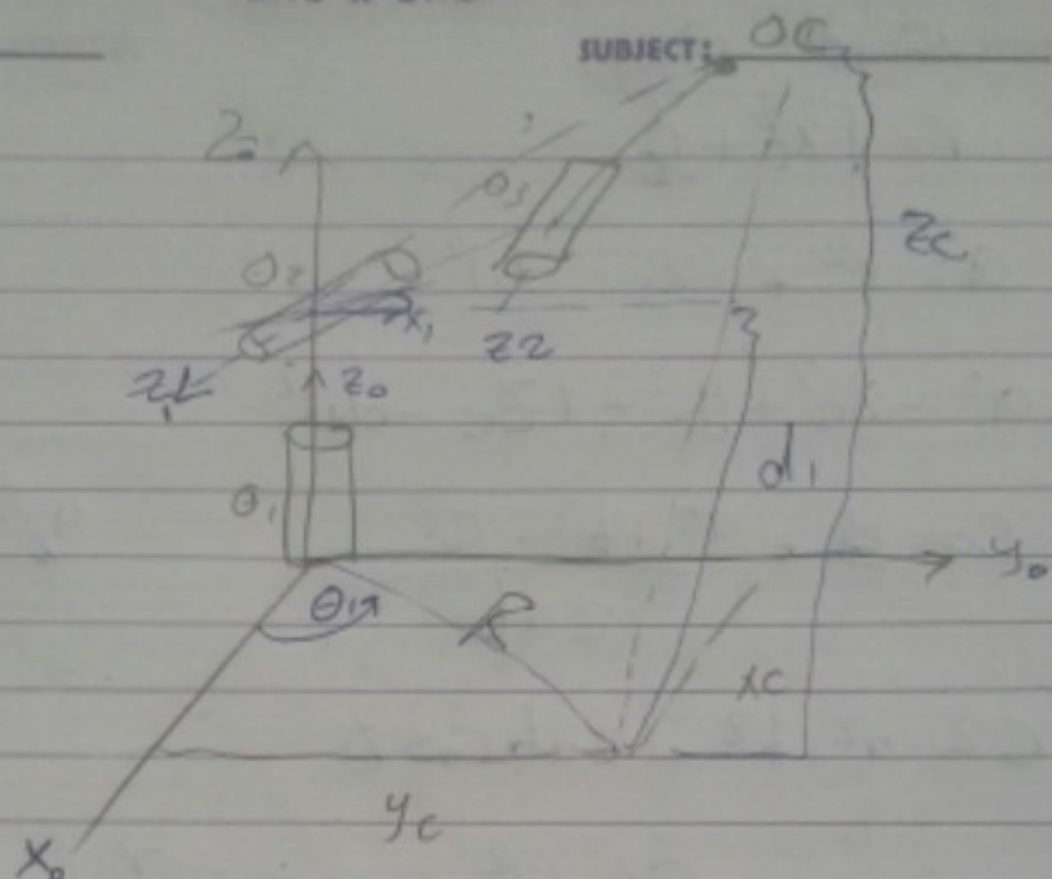
$$\dot{O}_b = \dot{O}_c + R_b d_4 \begin{bmatrix} \cdot \\ \cdot \\ 1 \end{bmatrix}$$

$$\dot{O}_c = \dot{O}_b - d_4 R_b \begin{bmatrix} \cdot \\ \cdot \\ 1 \end{bmatrix} \rightarrow (1)$$

O_{xc}, y_c, z_c

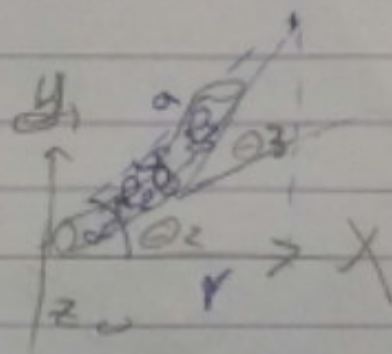
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$\theta_1 \rightarrow$ هي الزاوية بين قاطع الـ manipulator على مستوى الأرض

$\theta_2 \rightarrow$



$$\tan \theta_1 = \frac{y_c}{x_c}$$

$$\theta_1 = \tan^{-1} \left(\frac{y_c}{x_c} \right) \rightarrow (2)$$

$$= \text{atan2}(y_c, x_c) \rightarrow \text{matlab}$$

بما أن الربع وسرعة الإشارات، بالتقدير الدائري

DATE: For Q2, Q3

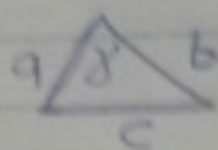
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$$r = \sqrt{x_c^2 + y_c^2}$$

$$a^2 = r^2 + (z_c - d)^2$$

$$a^2 = x_c^2 + y_c^2 + (z_c - d)^2$$

$$\cos \delta = \frac{a^2 + b^2 - c^2}{2ab}$$

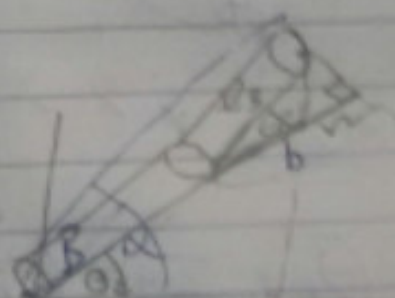


$$c^2 = a^2 + b^2 - 2ab \cos \delta$$

$$\cos \delta = \frac{l_2^2 + l_3^2 - c^2}{2l_2 l_3} \rightarrow c^2 = a^2$$

"Cosines Law"
based on adjacent

$$\theta_3 = 180 - \delta$$



$$\cos \theta_3 = \frac{l_2^2 + l_3^2 + x_c^2 + y_c^2 + (z_c - d)^2}{2l_2 l_3}$$

$$b = l_3 \cos \theta_3$$

$$B = \cos^{-1} \left(\frac{l_2 + l_3 \cos \theta_3}{l_3} \right)$$

$$(x_c^2 + y_c^2 + z_c^2 + d^2 - 2z_c d)$$

((((HMM)))

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$$\theta_2 = \tan^{-1} \frac{z_c - d_1}{\sqrt{x_c^2 + y_c^2}} - R$$

\propto

$${}^0R_6 = {}^0R_3 {}^3R_6$$

$${}^3R_6 = ({}^3R_3)^{-1} {}^0R_6$$

$\hookrightarrow \theta_4, \theta_5, \theta_6$
 using Euler parametrization.

substitute by $\theta_1, \theta_2, 3$